A Diagram to Facilitate the Study of External Ballistics.

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[Plate 5.]

The semi-graphical method about to be described enables solutions of many dynamical problems of a practical nature to be obtained with ease and rapidity. In particular, useful solutions can be found of problems relating to motion in a resisting medium in cases where the resistance is given as a function of the velocity which cannot be expressed by any simple algebraic form. In this paper the method is applied to the investigation of the motion of a projectile. Some of the curves obtained can be plotted directly from the ballistic table, but none of the curves are found in this way. They are derived by simple graphical processes from a primitive curve which represents the results of numerous experiments, reduced to give the resistance of a standard projectile, fired under standard conditions, as a function of the velocity. The curve, which is the basis of the diagrams given in the paper, is plotted from data given in the official 'Text-book of Small Arms,' 1909.

From the educational point of view the method about to be described has the advantage that the essential scientific principles relating to the motion of a projectile can be taught rapidly, and without reference to the laborious methods of analysis by means of which the ballistic tables are derived from the experimental data. As will be seen, other functions, besides those given in the ballistic tables, can be plotted with ease, because elimination is done by mere projection with set-square and T-square. By the drawing of a ballistic diagram a student of gunnery becomes familiar with the general relations existing between the different functions involved; in fact the set of curves constituting the ballistic diagram will probably, when drawn by himself, persist as a permanent mental picture of reference which cannot but be useful to a gunnery officer.

Although the ballistic table enables ranges to be calculated to the fraction of a foot in say 20,000 yards, and the time of flight to be calculated to a small fraction of a second (times are given to three places of decimals in the official Ballistic Tables) yet the tables cannot ensure a greater accuracy than the data from which they are derived, and it is probable that in many cases solutions of problems obtained directly from the ballistic diagram will have an order of accuracy as great as that of the primitive data, and certainly greater than the order of accuracy of the ballistic constant, since this is influenced by accidental variations of many kinds quite apart from the variation of muzzle velocity produced by variations in the quality of the propellant and the gravimetric density of the charge. In the final result, accuracy will of course depend upon the personal skill of the gunner and will be independent of tables, diagrams, and methods of analysis, but progress towards this personal skill may be made easier through a general knowledge of the curves of a ballistic diagram and the way in which they are derived from a primitive curve representing the air resistance as a function of the velocity.

Referring to fig. 1, curve No. 1 shows the resistance, p, of a projectile, 1 inch diameter, with a nose of standard shape, leaving the bore of the gun with a standard steadiness and moving through air of a standard density. When these conditions are varied, R, the resistance to motion, is stated in the official gunnery text-books to vary directly as d^2, the square of the diameter; directly as k, the coefficient of shape; directly as σ, the coefficient of steadiness; and directly as τ, the coefficient of tenuity of the air. That is to say

$$\mathrm{R} = k\sigma\tau d^2 p \text{ pounds.}$$

k, the coefficient of shape, depends upon the smoothness of the projectile and the shape of the head. In the official text-books it is stated that k varies from 2, in the case of flat-headed projectiles, to 0·8 for magazine-rifle bullets, with intermediate values of 1·7 for spherical shot and 0·95 for modern projectiles.

σ, the coefficient of steadiness, is normally less than unity, but may be greater than unity when the inner gun tube is worn or the projectile imperfectly centred or rotated.

τ, the coefficient of tenuity, is the density of the air expressed in terms of a standard density. This standard density was taken equal to 534·22 grains per cubic foot in Bashforth's original experiments. The value of this coefficient depends upon the temperature of the air and upon the height of the barometer.

The equation of motion through the air, of a projectile weighing w pounds, is now

$$-\mathrm{R} = k\sigma\tau d^2 p = \frac{w}{g}\frac{dv}{dt},$$

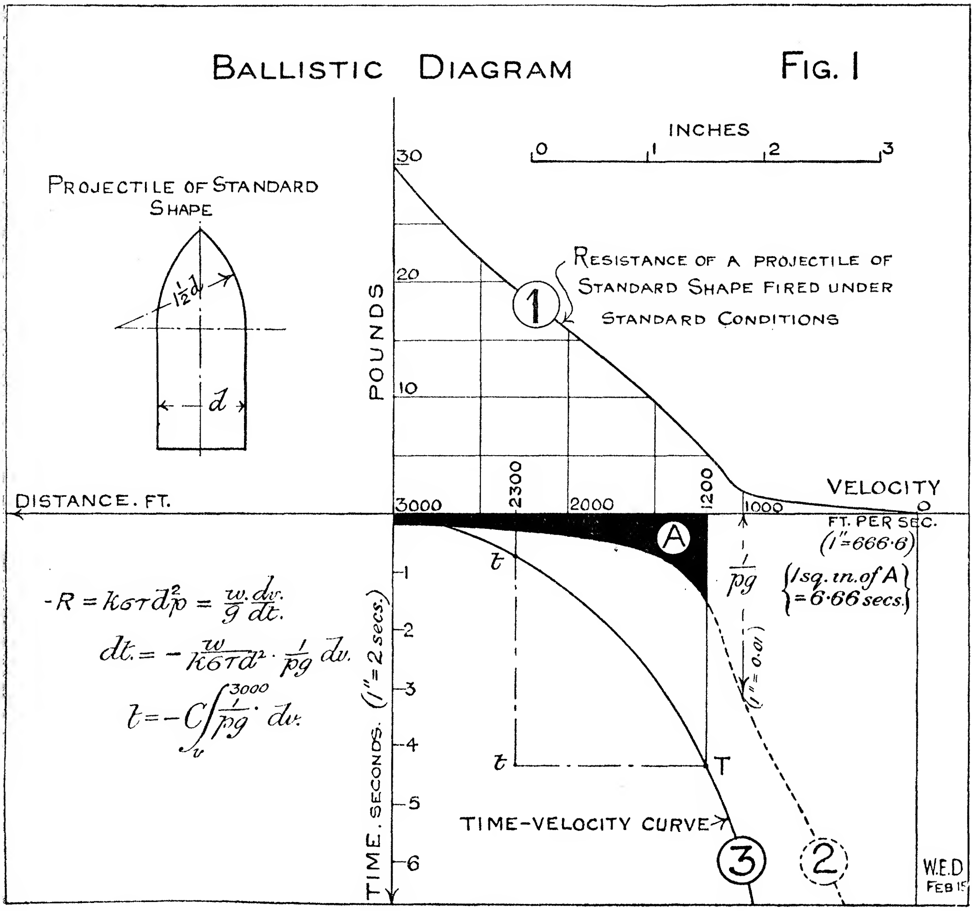
which gives

$$dt = -\frac{w}{k\sigma\tau d^2} \cdot \frac{1}{pg} \cdot dv = -\mathrm{C}\frac{1}{pg}\,dv. \tag{1}$$

The quantity $w/k\sigma\tau d^2$, which is represented by C, is constant for a particular projectile and gun for a particular state of the air, and it is usually called the ballistic coefficient of the projectile. The quantity $k\sigma\tau$ is called the coefficient of reduction. Integrating (1)

$$t = \mathrm{C}\int_{v}^{3000} \frac{1}{pg}\, dv.$$

This expression cannot be integrated directly because p is an arbitrary function of the velocity and falls under no standard form. The integral,


BALLISTIC DIAGRAM FIG. I

however, represents the area of a curve whose ordinates are $1/pg$. This curve is plotted below the velocity axis in fig. 1 and is marked 2. Points on it are quickly found by the aid of the slide-rule. The magnitude of p corresponding to a particular velocity is scaled off Curve 1 and the quotient $1/pg$ is calculated, and then, selecting arbitrarily any convenient scale, this quotient is plotted vertically downwards against the velocity. The black area marked A represents the value of the integral between the limits, $v = 3000$ and $v = 1200$. Selecting any convenient scale, this area is set down from the velocity axis along the ordinate through $v = 1200$, fixing thereby the point T on the time-velocity curve. The area represents the time taken for the velocity to fall from 3000 to 1200 feet per second.

The number of seconds represented by the area A depends upon the scales selected for the velocity and for the function $1/pg$. Thus if 1 unit of length measured along the velocity axis represents v feet per second, and 1 unit of length measured vertically downwards represents n $1/pg$ units; then 1 unit of the area A represents vn seconds. In the drawing of a diagram the most convenient unit of length to use is 1 inch.

For example, in fig. 1, 1 inch along the velocity axis represents in the original drawing 666·6 feet per second and 1 inch vertically downwards to curve No. 2 represents 0·01 $1/pg$ units. Therefore, 1 square inch of the area A represents on the original drawing $0{\cdot}01 \times 666{\cdot}6 = 6{\cdot}66$ seconds. The time represented by the area A is plotted in the original drawing to the arbitrarily chosen scale 1 inch = 2 seconds.

Areas like A corresponding to a series of ordinates taken at suitable intervals apart plotted vertically downwards from the velocity axis determine a sufficient number of points through which to draw the curve. Curve 3 is drawn through points found in this way.

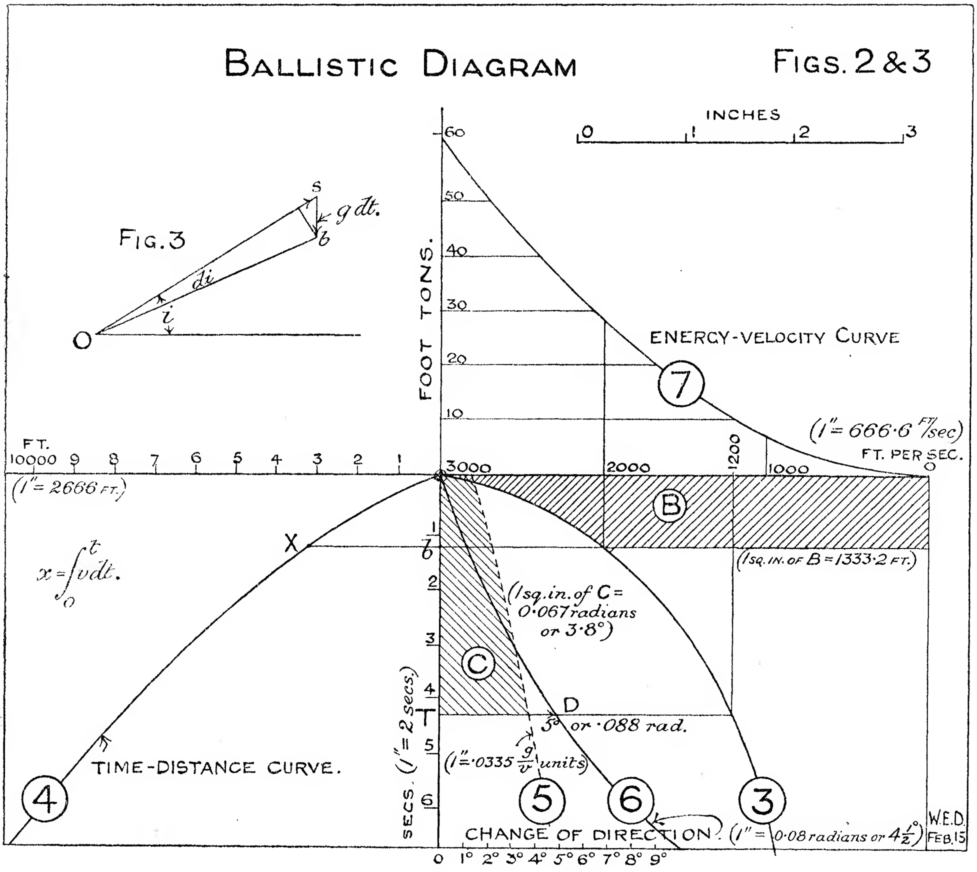
The time required for the standard projectile to change from any given velocity to any other velocity is found by measuring on the time scale the difference in the lengths of the ordinates to Curve 3 drawn through the given velocities. For example, the time taken by the standard projectile to fall from a velocity of 2300 feet per second to 1200 feet per second is represented by the length tt in fig. 1. The time corresponding to this length is called the diagram time. The actual time occupied by a projectile whose ballistic coefficient is C in changing from 2300 to 1200 feet per second is then $C \times tt$. Generally

Actual time = Diagram time × Ballistic coefficient.

The next curve to be found is one that gives the distance travelled by the standard projectile in terms of the time. Since

$$\frac{dx}{dt} = v, \qquad x = \int_0^t v\,dt. \tag{2}$$

Before this can be integrated the velocity v must be known in terms of the time. This is given by the curve just found and marked 3 in fig. 1. The distance travelled by the standard projectile during a change of velocity from 3000 to 2000 feet per second is represented by the area marked B in fig. 2. The time-velocity curve No. 3 is redrawn in fig. 2 in order that its integration with regard to t may be exhibited free from the lines used to obtain it in fig. 1. Carrying through to the left the ordinate which marks the limit of integration up to 2000 feet per second, the distance *b*X is set off to represent the area B. *b*X then represents the distance travelled by the standard projectile during the time O*b*, and equally the distance travelled whilst the velocity falls from 3000 to 2000 feet per second. X is a point on the time-distance curve marked 4. Integrating the time-velocity curve in this way step by step points are obtained in sufficient number to define Curve 4.



The distance represented by the area B depends upon the time scale and the velocity scale which have already been chosen. Thus 1 inch represents 666·6 feet per second; 1 inch represents 2 seconds. Therefore 1 square inch of the area B represents 1333·3 feet. Distance is set out to the arbitrarily chosen scale 1 inch = 2666 feet.

Consider now the change of direction in the trajectory caused by the action of gravity. Let O*s* (fig. 3) be the velocity of the projectile at a point in the trajectory. After the small interval of time dt the velocity will be changed to Ob owing to the addition of the velocity $g\,dt$ due to the action of gravity. Let di be the change in direction. Then from the figure

$$v\,di = g\,dt \cos i.$$

It is assumed in the official gunnery text-book that within elevations of 15 degrees $\cos i$ may be put equal to unity. With this approximation

$$i = \int_0^{\mathrm{T}} \frac{g}{v}\,dt. \tag{3}$$

Before this can be integrated g/v must be known as a function of the time t. But v as a function of the time t is given by curve No. 3. Therefore take a series of values of the time and plot the auxiliary curve No. 5 by measuring off the velocity corresponding to a given time, and then by means of a slide-rule calculating the value g/v. Then integrate curve No. 5 with respect to the time. Thus the area C represents the change of direction in the time OT. This area plotted to any convenient scale gives the point D, and other points found in the same way determine the curve No. 6.

The change of direction represented by the area C depends upon the time scale and the scale on which the values of g/v are plotted. In the figure the original scales were: 1 inch horizontally = 0·0335 g/v units, 1 inch downwards represents 2 seconds. Therefore 1 square inch of the area C represents on the original drawing a change of direction of 0·067 radian or $0{\cdot}067 \times 57{\cdot}3 = 3{\cdot}8$ degrees. And the scale on which change of direction is plotted is approximately 1 inch = $4\frac{1}{2}$ degrees or 0·088 radian.

The change of direction is integrated in relation to the diagram time. The change of direction given by the ordinates to Curve 6 measured horizontally from the time axis is, therefore, the diagram change. The actual change of direction is found by multiplying this diagram change by the ballistic coefficient of the projectile.

Curve No. 7 shows the linear kinetic energy in foot-tons stored in the projectile per pound weight of the projectile. The striking energy corresponding to any terminal velocity can be obtained from this curve. The actual striking energy in foot-tons is the energy found from the curve multiplied by the weight of the projectile in pounds.

The group of curves shown in fig. 4 constitute the ballistic diagram.

Let V be any given muzzle velocity, and let v be the final velocity. Through V and v draw vertical lines to cut the time-velocity curve No. 3, and through these points draw horizontal lines to cut the time-distance curve No. 4 in X and x; finally draw vertical lines through X and x to cut the distance axis in S and s. Then Ss is the distance travelled by the projectile whose ballistic coefficient is unity during the time that the velocity changes from V to v, and Tt is the corresponding time of flight.

With the limitations of elevation and altitude assumed in low-angle fire, it is assumed in the official gunnery text-book that the time occupied by the projectile in reaching the highest point in the trajectory is equal to the time of its descent to the level from which it was fired. Equal times of ascent to, and descent from, the vertex of the trajectory involve the condition that the vertical velocity is so low that the vertical component of the frictional resistance of the air is negligible. Bisect Tt in L. Project the point L horizontally and vertically to V_0 and S_0. Then SS_0 is the horizontal distance corresponding to the greatest altitude, and the point V_0 gives the horizontal velocity at the vertex.

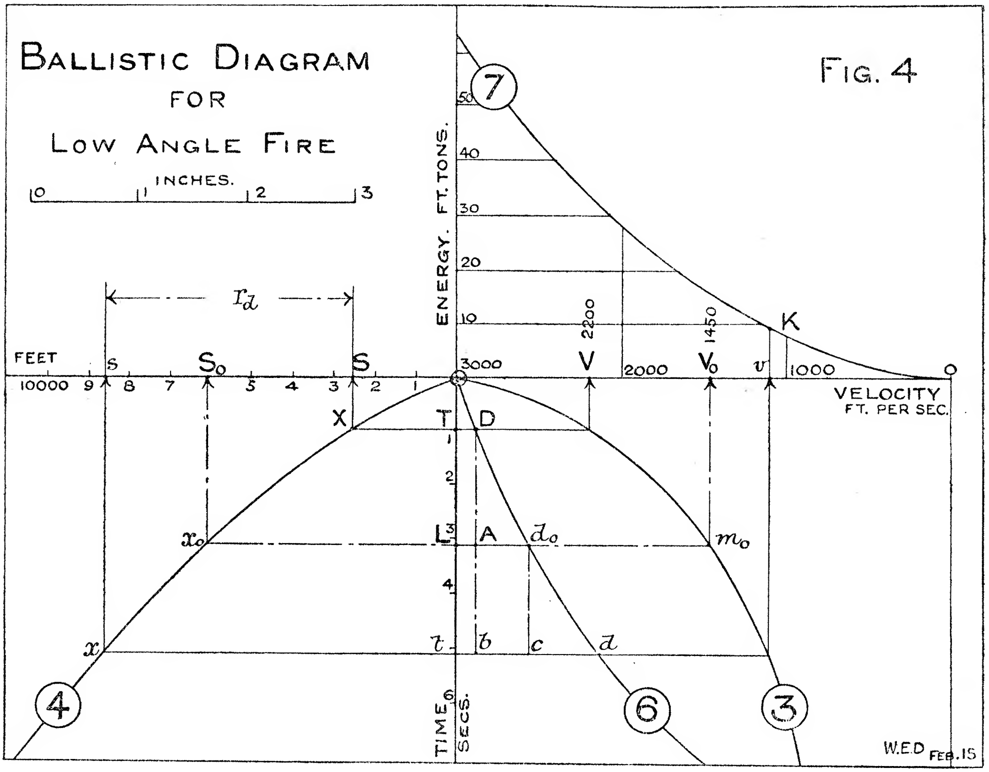


Fig. 4

Consider, now, the intercepts cut off by the change of direction curve No. 6; td—TD is the change of direction during the flight. Through D draw the line Db parallel to the time axis, and through d_0 draw d_0c parallel to the time axis. Then bd represents the change of direction of the standard projectile fired under standard conditions during the time of flight

Tt. Ad_0 represents the change of direction up to the vertex, and this is therefore the angle of elevation of the gun or its "tangent elevation." cd represents the change of direction from the vertex to the level from which the gun was fired, and this is therefore the angle of descent. The angles Ad_0, cd, measured from the diagram, are to be multiplied by the ballistic coefficient of the projectile to find the actual angles.

These geometrical properties indicate the solution of many gunnery problems. For example, assume that the muzzle velocity is given, together with the value of the ballistic constant C and the range r. The time of flight, the terminal velocity, the striking energy, the angle of elevation, the angle of descent, the time to the vertex, the horizontal distance to the vertex, and the horizontal velocity at the vertex, can all be found from the diagram after drawing a few lines with T-square or set-square. In this particular problem the distance apart of the horizontal lines through T and t is fixed by the given range.

First find the diagram range corresponding to the standard projectile fired under standard conditions. The diagram range r_d is given by

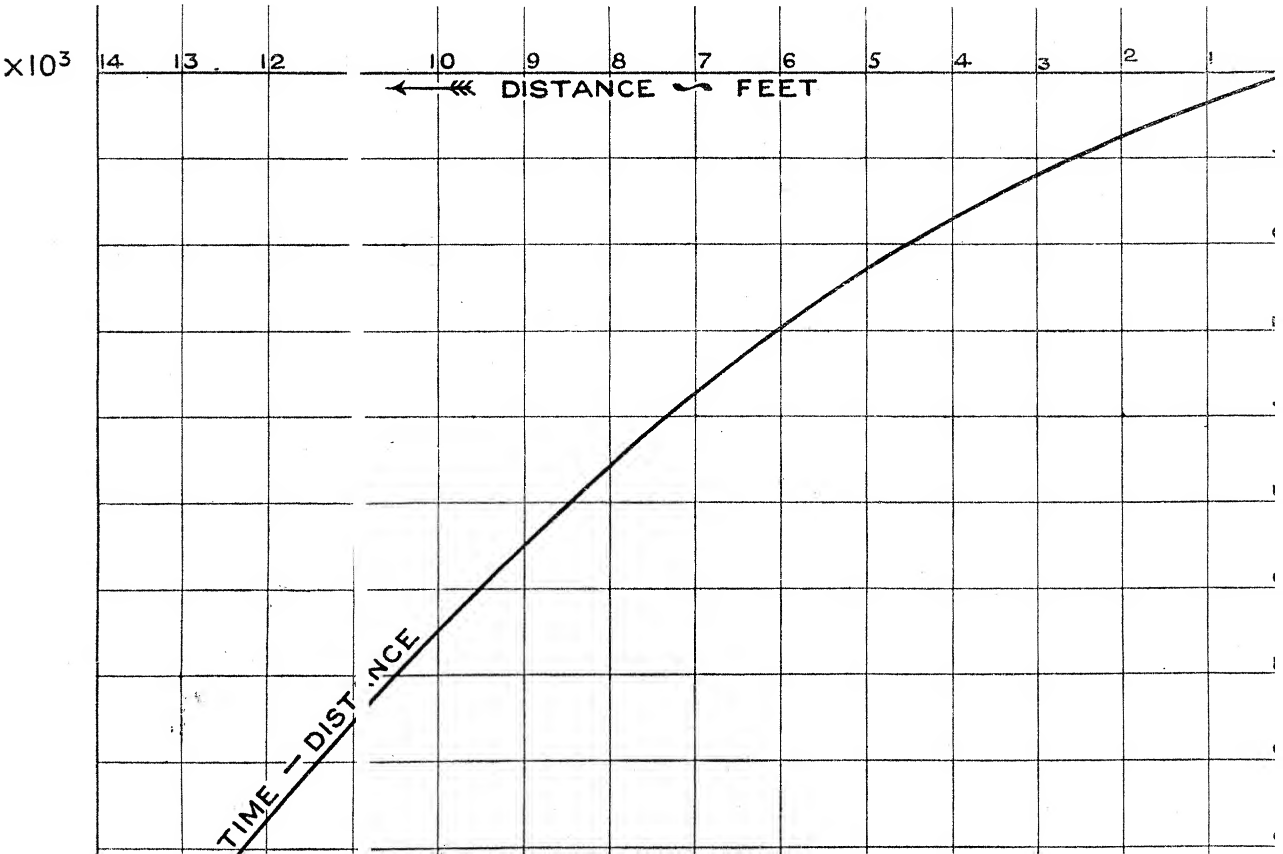
$$r_d = \frac{\text{actual range}}{\text{C}}.$$

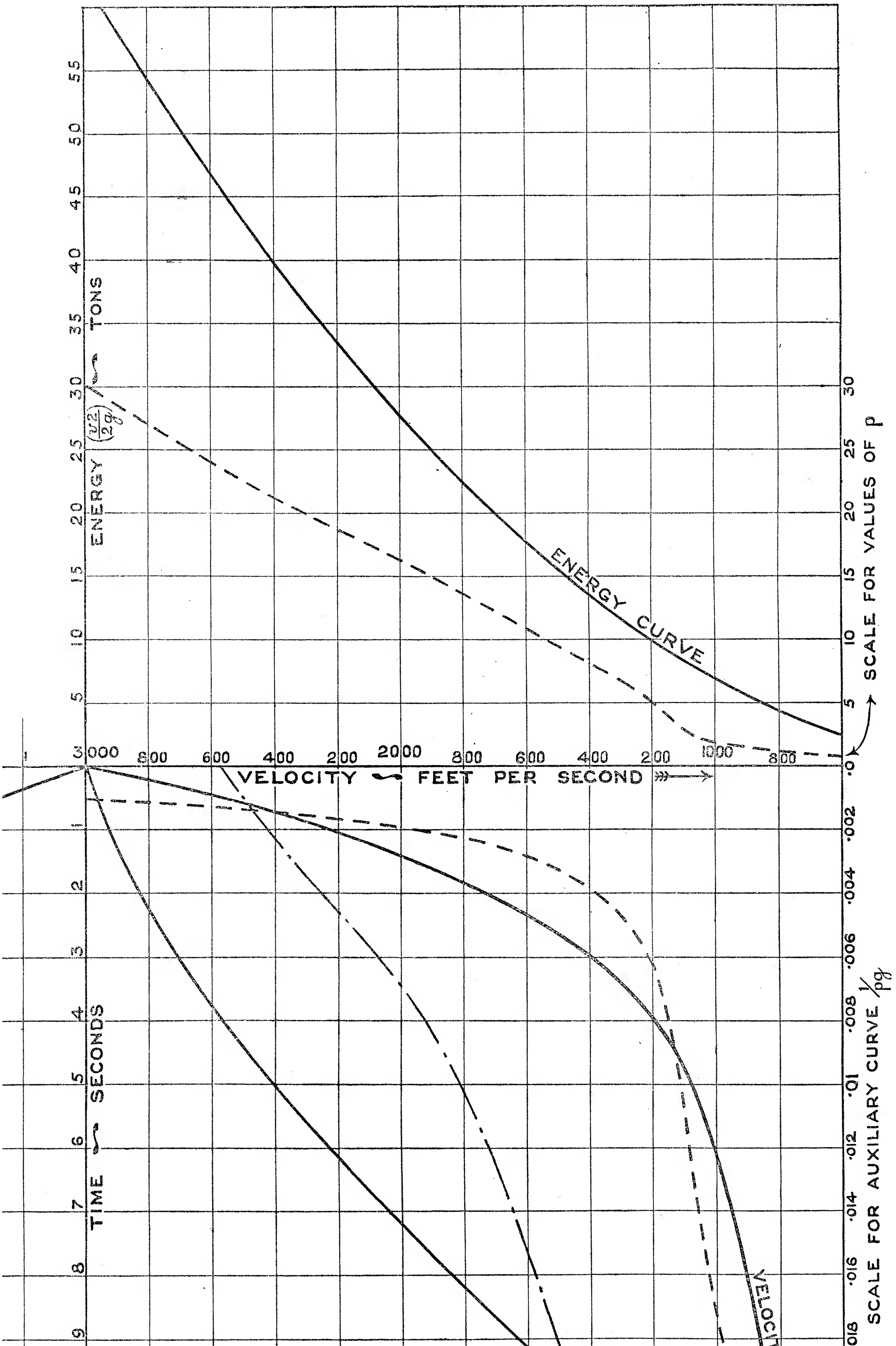
Project the given muzzle velocity V downwards to curve No. 3, and then horizontally to curve No. 4, and finally upwards to S on the distance axis. These ordinates and the horizontal connecting them form the datum lines for the given initial velocity V. From the point S set out the diagram range r_d. Assume that this range is represented by the distance Ss. Project the point s to curve No. 4, and then horizontally to curve No. 3, and then vertically to cut the horizontal velocity axis in v, and the kinetic energy curve No. 7 in K. Finally bisect Tt in L, and through L draw a horizontal to cut the three curves in x_0, d_0, and m_0, and through these points draw ordinates to the axes. Then, remembering that C is the ballistic constant,

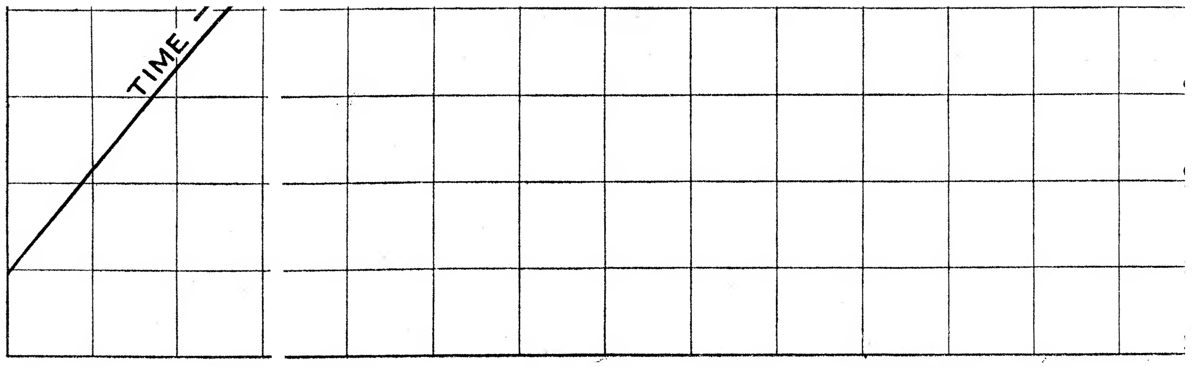
Time of flight = Tt × C seconds.
Terminal velocity = v feet per second.
Striking energy = vK × W foot tons. (W = weight of projectile in pounds.)
Angle of elevation = Ad_0 × C degrees.
Angle of descent = cd × C degrees.
Time to the vertex = $\frac{1}{2}$Tt × C seconds.
Horizontal distance to the vertex = SS_0 × C feet.
Horizontal velocity at the vertex = V_0 feet per second.

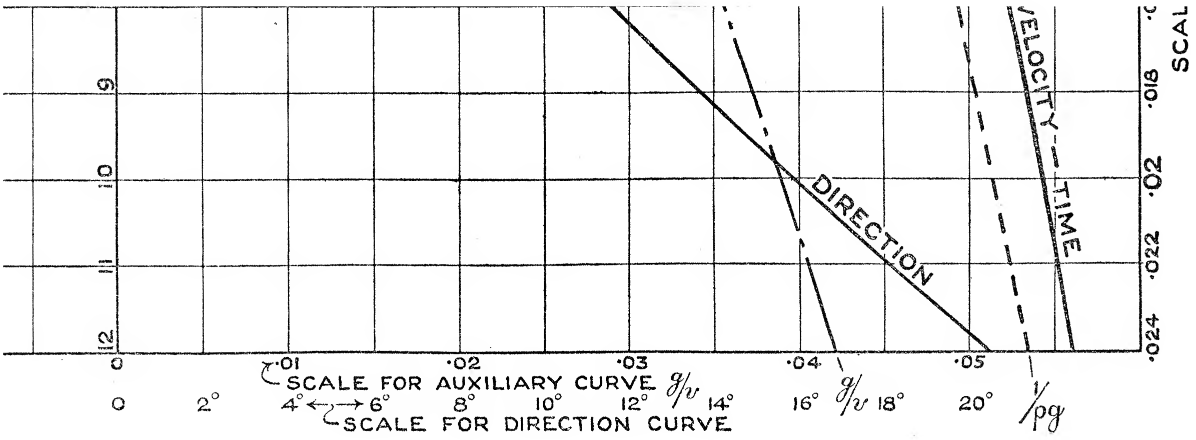
Dalby.

BALLISTIC DIAGRAM
FOR
DIRECT FIRE




55
50
45
40
35
30
25
20
15
10
5
ENERGY $\left(\frac{v^2}{2g}\right)$ ~ TONS
3000
800
600
400
200
2000
800
600
400
200
1000
800
VELOCITY ~ FEET PER SECOND
ENERGY CURVE
SCALE FOR VALUES OF ρ
30
25
20
15
10
5
·0
·002
·004
·006
·008
·01
·012
·014
·016
·018
SCALE FOR AUXILIARY CURVE $^1/_{\rho g}$
1
2
3
4
5
6
7
8
9
TIME ~ SECONDS
VELOCI


TIME —


9
10
11
12
DIRECTION
VELOCITY
TIME
SCAL
·018
·02
·022
·024
0
·01
·02
·03
·04
·05
SCALE FOR AUXILIARY CURVE g/v
0
2°
4°
6°
8°
10°
12°
14°
16°
g/v
18°
20°
$1/pg$
SCALE FOR DIRECTION CURVE

It will be understood from this problem that many similar problems in gunnery can be solved by the aid of this diagram with an accuracy probably near enough for most practical purposes.

An accurately drawn ballistic diagram accompanies this paper and is reproduced in the folded plate, reduced to slightly less than half size. It may be used in the way exemplified by fig. 4 for the solution of problems of gunnery relating to direct-angle fire. The scales to which the original of this diagram is drawn are:—

Velocity, 1 inch = 200 feet per second.
Distance, 1 inch = 1000 feet.
Time, 1 inch = 1 second.
Energy, 1 inch = 5 foot-tons.

Auxiliary curves:—

$1/pg$, 1 inch = 0·002 unit.
g/v, 1 inch = 0·005 unit.

Surface Friction: Experiments with Steam and Water in Pipes.

By Cecil H. Lander, M.Sc., A.M.I.C.E.

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During the past hundred years much work, both theoretical and experimental, has been carried out with a view to determining the character of the laws governing the resistance to tangential motion between solid surfaces and liquids or gases. A general relation between the dimensions of the surface, the velocity, the density of the fluid, and its viscosity had been surmised as a consequence of the laws of motion by Stokes, Helmholtz, and Osborne Reynolds, but it was left to Lord Rayleigh[*] to show, from the principle of dynamical similarity, that the phenomena involved could be expressed definitely by a simple mathematical formula.

The laws governing the friction between solid surfaces and water have formed the subject of experimental investigations by Froude, Osborne Reynolds, Darcy, etc., whilst the parallel case of the resistance to motion between solid surfaces and perfect gases has occupied the attention of Zahm, Brix, Stockalper, and others. Practically all these investigators devoted their energies to experimental determinations of the friction in the medium which they employed, and it was not until the subject was taken

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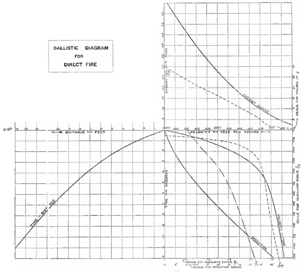
[*] 'Phil. Mag.,' p. 321 (1899). 'Advisory Committee for Aeronautics Report,' 1909–10, p. 38.


BALLISTIC DIAGRAM
FOR
DIRECT FIRE
ENERGY CURVE
ENERGY
SCALE FOR VALUES OF p
×10³
DISTANCE — FEET
3000
2000
VELOCITY — FEET PER SECOND
TIME — DISTANCE
TIME — SECONDS
DIRECTION
VELOCITY — TIME
SCALE FOR AUXILIARY CURVE
SCALE FOR AUXILIARY CURVE
SCALE FOR DIRECTION CURVE